RECURSIVE CAUSAL CONVOLUTION

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Abstract

A new method is presented for implementing discrete numerical convolution in passive circuit simulation which achieves high efficiency by using a recursive formulation. The technique starts with an FIR response obtained by a conventional convolution method and obtains an IIR filter using Padè approximants in the z-domain. The resulting IIR filter is tested for stability using the Lehmer-Schur algorithm and for accuracy by comparing its impulse response to that of the original FIR filter.

Introduction

There is a growing interest in efficient numerical techniques for the representation of arbitrary lossy distributed systems within a nonlinear transient simulator. There are two main procedures for the simulation of passive distributed circuits in the time domain, namely, convolution and lumped element approximation. Both involve approximating the original frequency function by either a series of exponential functions, in the case of convolution, or a rational function, for the lumped element approximation. The causal convolution technique[1], which is a convolution based method, provides a general methodology to obtain a causal and stable response of a general passive circuit, although it may suffer from limitations in terms of simulation efficiency. AWE methods [2-3] implement the lumped element approximation by employing Padè Approximants to obtain a rational function that approximates the original frequency function. Although the AWE methods are very efficient when implemented recursively [3], they suffer also from some drawbacks since the Padè approximation requires a series representation of the original frequency function in the Laplace domain and may result in unstable and/or uncausal rational functions.

The procedure presented here is based on the causal convolution formulation[1] which allows a frequency function to be modeled as a causal FIR (Finite Impulse Response) filter. Using a recursive representation, it is shown that greatly improved numerical efficiency may be achieved by transforming from an FIR to an IIR (Infinite Impulse Response) representation. The order of the resulting IIR filter is increased until stable implementation is achieved. The IIR filter is also tested for accuracy where the maximum error relative to the maximum FIR filter coefficient, is below a suitable bound. The technique will only fail if no suitable IIR filter is found that is both stable and has the same impulse response as the FIR filter with fewer time steps or a smaller number of coefficients. This approach offers a reliable and potentially highly-efficient method for simulation of distributed circuits within non-linear time domain simulators.

IIR representation from FIR

As shown in [1], an FIR filter can be used to approximate the time-domain representation of an original frequency function. This function can be either represented in analytical or sampled data form, both of which can produce equally spaced data points. Numerically, this is done over a suitable frequency range and within some error bound. The FIR is represented in the time domain as a series of weighted Dirac Delta functions is,

$$f(t) = \sum_{i=0}^{N} h_i \delta(t - i\tau) \tag{1}$$

which is causal by definition and stable considering the condition that the coefficients after N are negligible. This response in the z-domain is,

$$f(z) = \sum_{i=0}^{N} h_i z^{-i} \tag{2}$$

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where the region of convergence of the series is $0 < \|z\| < \infty$. This representation is unique in the sense that for every two functions that have the same series representation in z, they will have the same impulse response. Therefore the main idea is to find a rational representation of f(z) which has the same series representation. The aim would be to reduce both the order of the f(z) function and the number of coefficients. This is the exact definition of the Padè approximants when applied at infinity [4]. The usual form of Padè's rational function taken at infinity is,

$$g(z) = \frac{\sum_{i=0}^{L} \hat{a}_i z^{-i}}{(1 + \sum_{i=1}^{M} \hat{b}_i z^{-i})} \tag{3}$$

This is very similar to the IIR filter in the z-domain which has the form

$$g(z) = \frac{\sum_{i=0}^{L} a_i z^{-i}}{(1 - \sum_{i=1}^{M} b_i z^{-i})} \tag{4}$$

Where $a_i = \hat{a}_i$ and $b_i = -\hat{b}_i$ with $a_0 = h_0$. The resulting IIR filter will use M+L+1 coefficients of h_i and will have the usual difference equation form,

$$y_n = \sum_{i=0}^{L} a_i x_{n-i} + \sum_{i=1}^{M} b_i y_{n-i} \tag{5}$$

When $M + L + 1 > N$ we can either augment the series with zeros to make sure that the infinite response of the IIR filter terminates or extend it in any particular fashion for cases where the original is known to have an infinite response, e.g. the S parameter of an open or short circuit. The procedure suggested by the above reasoning is to start with the lowest possible order to obtain an IIR filter function and then increase the order, testing repeatedly for stability and accuracy until we either reach a suitable IIR filter or the order is increased beyond some limit. The limit is devised based on implementation specifics; a memory limit would limit the time steps to $N/2$ since above this the space required to store previous time points is larger than the original FIR filter, and a complexity limit would require that $M + L + 1 < N$, since this would reduce the number of convolution operations required per time step in a transient analysis.

Stability and Accuracy tests

An IIR filter is unstable if its z domain form has at least one pole outside the unit circle in the complex plane. As a result a suitable test would be to check whether the denominator has zeroes outside the unit circle. To that effect we utilize the Lehmer-Schur algorithm[5] which tests if a polynomial has zeroes inside the unit circle. Thus before applying the algorithm we use

$$z \mapsto \frac{1}{w} \tag{6}$$

to map the outside of the unit circle to its interior and then apply the test on f(w). This test is $O(n^2)$ where n is the order of the polynomial and is analogous to the Hurwitz procedure[6] in the Laplace Domain. To obtain the Impulse response for an IIR filter one can either evaluate the difference equation or obtain the Laurent series around infinity i.e. the z-transform in series representation. Once the time response is found it is compared to the original FIR response.

Examples

The IIR filter obtained above will give the exact analytical form for the S parameters of an unmatched lossless or lossy distortionless [7] transmission lines. Such lines will have FIR response as in Fig.1.

However, when implemented as an IIR filter it will have just one time step for S11 and two time steps for S12 with a (stable) pole zero distribution as in Fig.2.

Tests on more general transmission lines including loss and dispersion indicate suitable forms of IIR filters would give large savings in the coefficients required for the IIR filter compared to the starting FIR filter as shown in Fig.3 and Fig.4 and accompanying Tables. For example, while for the dispersive lossy line 97 convolution steps were needed in conventional methods, only 17 are needed in the recursive formulation.

Conclusion

The method proposed here will work with either analytical or sampled frequency functions. By using a recursive formulation, it offers significant computaional advantages over standard convolution techniques, and retains the essential requirements of causality and stability.

References

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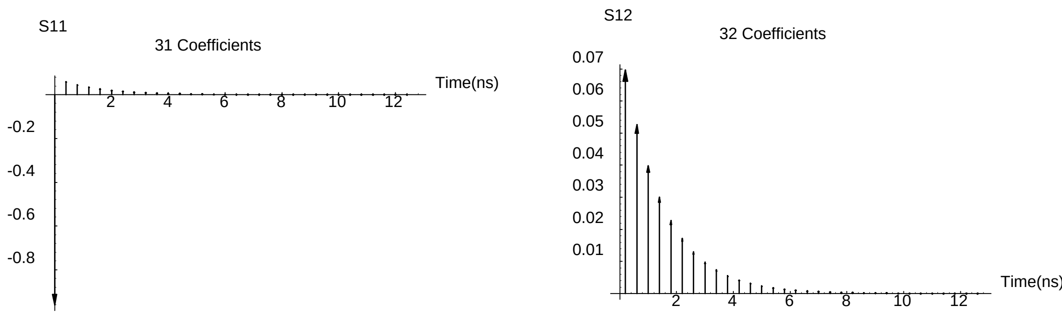

Figure 1: FIR impulse response of a lossy distortionless 1Ω line with 50Ω termination which has quarter wavelength at 2.5GHz and $\alpha = 10$ nepers/meter. S11 on the left, S12 on the right

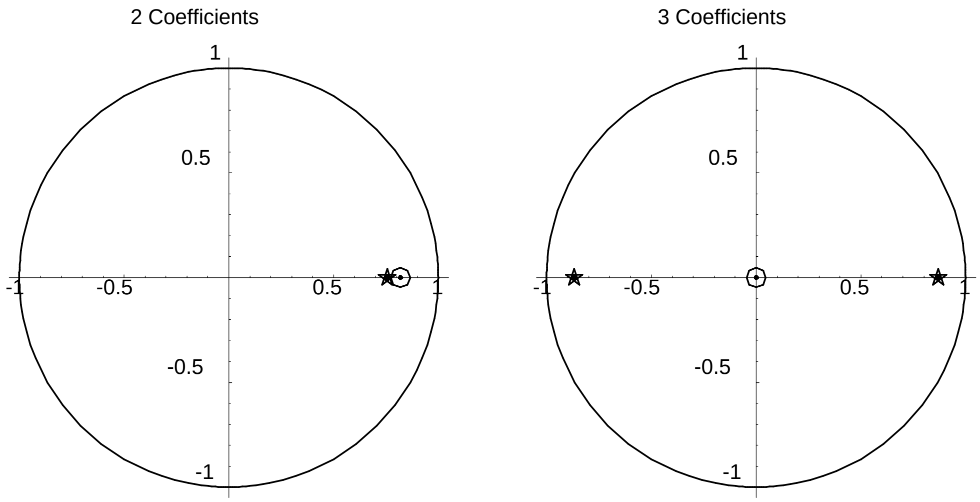

Figure 2: Pole zero positions of the lossy distortionless line above. S11 on the left, S12 on the right. $\star$ for a pole, $\circ$ for a zero.

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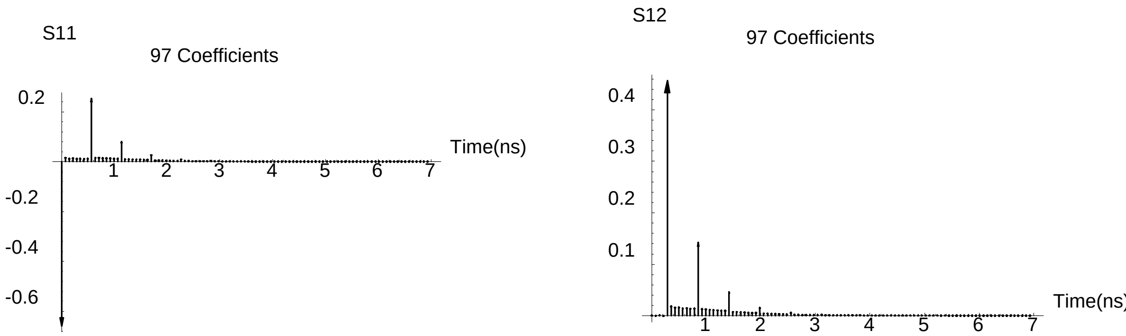

Figure 3: FIR impulse response of a lossy dispersive line with 50Ω. S11 on the left, S12 on the right

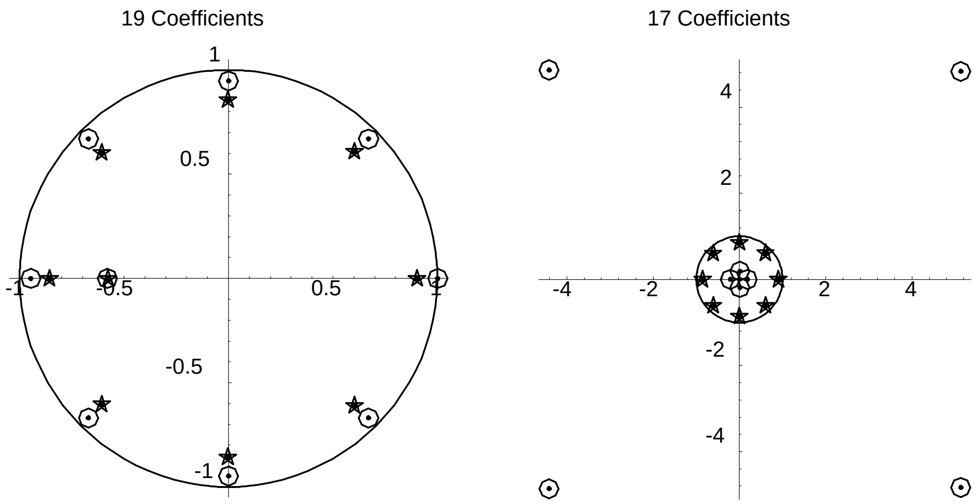

Figure 4: Pole zero positions of the lossy dispersive line. S11 on the left, S12 on the right. $\star$ for a pole, $\circ$ for a zero.

Function	Filter	No. of Coefficients	No. of Taps in filter	IIR Memory Advantage	IIR Complexity Advantage	IIR speed-up Factor
S11	FIR	31	31	15.5	10.3	9.6
	IIR	3	1			
S12	FIR	32	64	16	8	7.3
	IIR	4	2			

Table 1: Comparison showing the advantages of IIR compared to FIR filters. Simulation performed on a specially modified SPICE simulator.

Function	Filter	No. of Coefficients	No. of Taps in filter	IIR Memory Advantage	IIR Complexity Advantage	IIR Speed-up Factor
S11	FIR	97	97	5.1	5.4	4.1
	IIR	19	9			
S12	FIR	97	97	5.7	6.1	4.9
	IIR	17	8			

Table 2: Comparison showing the advantages of IIR compared to FIR filters. Simulation performed on a specially modified SPICE simulator.

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